It is clear that the adjustment will depend in part on the position of the lighthouse, especially on its height above sea-level, and that a system of lenses put together without any reference to this was bound to be wrong.

The results were entirely satisfactory, and the Whitby light when reconstructed gave admirable results.

A good deal of correspondence followed with some members of the Commission as to the form of lamp and the best height for the principal focus of the system above the wick, and as time went on various other improvements were introduced; but Mr. Chance's position was now assured, and is evidenced by the long list of splendid lights we owe to him.

One of the improvements worked out in collaboration with Mr. Thomas Stevenson is the dioptric mirror, whereby the rays which leave the lamp at the back are totally reflected by suitable curved prisms and issue in the direction in which the light is required to travel.

The whole book, however, is most interesting, and forms a striking illustration of the application of science to industry. Mr. Chance realised the need of this, and his success was the consequence.

His contributions to the mathematical side of the subject are summed up in two papers read before the Institution of Civil Engineers in 1867 and 1879. The first deals with lighthouses in general; its value as a reprint is, however, impaired by the omission of the careful figures by which it was illustrated; in the second, the question of the application of the electric light to lighthouses is considered. Electric light was employed at the South Foreland in 1872 and at the Lizard lighthouse in 1878. The apparatus in the latter case was designed by John Hopkinson, who on Mr. Chance's retirement became scientific adviser to the firm.

R. T. G.

THE INFINITIES OF MATHEMATICS.

Die Grundsätze und das Wesen des Unendlichen in der Mathematik und Philosophie. Von Dr. Phil. Kurt Geissler. Pp. viii + 417. (Leipzig: Teubner, 1902.) Price 14 marks.

EVERY serious inquiry leads, sooner or later, to metaphysics, and thus to antinomies which no merely logical process can reconcile. The pure mathematician is one of the first to reach this conclusion, because his methods are mainly logical, and the notions with which he deals are few and abstract. Why is it, then, that (as a rule) he regards the philosopher with a mixture of pity and disdain, and rarely takes part in any strictly metaphysical discussion? Each is vitally concerned with number, space and time; why do the conclusions of the one appeal so little to those of the other? Leaving the philosopher to answer for himself, we may endeavour to construct the mathematician's apology.

It is mainly that, while he reaches the fundamental paradoxes as soon as the metaphysician, his attitude towards them is different. As it seems to him, the philosopher, after an imperfect analysis, tries to save the situation by a still more imperfect transcendental synthesis. To swamp all distinctions in the Absolute, while assuring us that the distinctions persist, is a childishly simple course, especially when adopted by someone who has a very vague conception of the distinctions which he proposes to abolish. Surely it is reasonable to examine our concepts as carefully as we can, to discover, if possible, which are simple and which are derived or composite. Until we do this, we have no right to say what are the ultimate logical inconsistencies, still less how we propose to reconcile them. The presuppositions of arithmetic and geometry have recently been analysed with great care, and definite results of primary importance have been obtained; the philosophical bearing of these conclusions is obvious, and henceforth no metaphysical theory that ignores them will be accepted by mathematicians. Difficulties remain, of course; some have emerged which were previously unsuspected; but at any rate the ground has been cleared of many merely sophistical paradoxes, and the real issues have been made clearer.

Dr. Geissler's book is rather pathetically disappointing; he has evidently tried to master modern critical theories, but has failed in the attempt. The whole arrangement of the work is unsatisfactory, starting as it does with a vague spatial intuition, and constantly mixing up arithmetical difficulties with those of geometry. In the forefront of all discussions of mathematical infinity must be put the notion of the arithmetical continuum; this, at any rate, is precise and definite. From it we get the concept of a continuous real variable, and thence can proceed to the differential and integral calculus treated by the method of limits. This involves the use of a fluent differential, but there are no serious logical difficulties. Dr. Geissler's attitude is anything but precise, and not always consistent; he appears to try to establish the existence of infinitesimals of different orders as actual entities, and this partly by geometrical considerations. In this region of thought geometrical intuition is wholly untrustworthy; and it is doubtful whether any satisfactory analytical theory can be constructed on the basis of what we may call fixed infinitesimals. It is certain, for instance, that in the arithmetical continuum there is no natural series of orders of infinitesimals. What is the precise nature of geometrical continuity, and how far it can be expressed by arithmetical means, is a very difficult question, upon which Dr. Geissler does not help to shed any light.

One important point the author does emphasise, though sometimes with more zeal than discretion. The terms *infinite* and *infinitesimal* have no precise meaning except in relation to a context and to certain presuppositions. Thus, in projective geometry, the statement that all points at infinity lie in a plane is a convenient summary of a set of facts about parallels; on the other hand, in the theory of algebraic functions, we assume that in the plane of the complex variable there is only a single point (not a line) at infinity. Each statement is true in its context, and out of its context it means nothing at all. If, with Dr. Geissler, we set off equal finite segments continually along a Euclidean straight line, we may assert the possibility of any

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number of finite segments at infinity; to enumerate them we require transcendent integers, but there is nothing illogical in the conception, provided that we use it consistently. But we must not criticise one conception by the results of another with which it is radically incompatible.

To show how weak the author's logic is, it is sufficient to refer to his discussion of the old fallacy of Achilles and the tortoise. Here it is established that an indefinite number of successive intervals of time can be found, for each of which the tortoise is ahead; and it is falsely concluded that the sum of these intervals of time tends to an indefinitely long period. Instead of pointing out this simple fact, Dr. Geissler argues that the division of the initial interval between Achilles and the tortoise is illegitimate! ("Man darf sich nicht einbilden, es liege schon im Wesen einer Strecke AB auf ihr in irgend einer Weise Strecken zu tragen.") In fact, his hankering for infinitesimals, in the sense of indivisibles, makes all his treatment of limits and convergence quite unsatisfactory.

It is a matter for real regret that Dr. Geissler has so completely failed to contribute anything of value to the discussion of his subject. The development of mathematics since the time of Kant has surely provided some new material for speculation; how long must we wait for a philosopher competent to deal with it? Even De Morgan failed to appreciate Rowan Hamilton's conception of algebra as the science of pure time; the truth of this idea (except, perhaps, for group-theory) is becoming daily more evident. But while analysis is thus practically reduced to a subjective construction, there are elements in geometry which refuse to be so assimilated. Not all mathematicians are geometers, but those who are will sturdily maintain that, in some sense or other, there are geometrical data which are not expressible in terms of arithmetic. The present tendency towards critical analysis may, we hope, be succeeded by renewed interest in pure geometry. Then, perhaps, something more may be done towards distinguishing its primary axioms.

The analytical doctrine of the infinite has been sketched in a very interesting manner by Dr. Hobson in his recent presidential address to the London Mathematical Society; this deserves to be widely read, because it presents the main discoveries of Dedekind, Cantor, &c., in a form which does not assume any advanced mathematical knowledge on the part of the reader.

G. B. M.

PRACTICAL PHYSIOLOGY.

Practical Physiology. By A. P. Beddard, J. S. Edkins, Leonard Hill, J. J. R. Macleod and M. S. Pembrey. Pp. xiv + 495 (London: Edward Arnold, 1902.) Price 15*s*. net.

THE aim of the authors of this text-book has been to provide medical students with a course of physiology which shall not only give them sufficient mental training—practical Chinese or Greek would do this—but also ensure that this training shall be of substantial use in their after medical career. In some respects, this aim has been carried out in an admirable manner. Many of the articles are clearly written and well illustrated, and some of the sections—notably those on circulation, blood gases and physiological optics—contain valuable information not readily accessible to the student in any of the existing text-books. Other parts hardly maintain this high level, and the chapters on muscle and nerve in particular are noticeably deficient, even allowing for the author's expressions of dissent in the preface, and it is curious to see a text-book of 495 pages in which only two are devoted to the description of the galvanometer and capillary electrometer together, and where three lines contain the major part of the information on the electromotive phenomena of the heart!

The book is divided into four parts; the first two of these, comprising the more elementary exercises, are, on the whole, very good. The labour involved in preparing new illustrations and tracings must have been considerable, and the authors are justly to be congratulated on the result of their exertions. In addition to the experimental exercises already noticed, the section on physiological chemistry merits favourable comment, and as a pleasing matter of detail, the drawings of crystals are particularly accurate.

There are certain places, however, which might be subjected to a little revision in the next edition. While the authors reject Von Fleischl's hæmoglobinometer on the ground of inaccuracy, the directions given for the use of the Thoma-Zeiss hæmocytometer will in practice lead all but the very careful student to far more serious errors of estimation. With a little skill, the method for the detection of albuminuria by heat could be carried out so as to show no trace of albumin even when it was present in considerable amount, as acid albuminate would readily be formed under the conditions recommended. "Ethylic acid," on p. 180, is a pretty obvious misprint for "ethyl alcohol." We think that the information on p. 72 is a little out of place, but perhaps this is a matter of opinion.

The "advanced" portion of the text-book is hardly so well written as the "elementary," though the articles on optics and on Haldane's methods for determining oxygen capacity and mass of the blood could hardly be surpassed. Here, moreover, is to be found the largest part of the "comedy of errors" which is inseparable from a first edition. Constantine was an Emperor who reigned at Constantinople; the alloy of manganese and iron used for thermoelectric work was not called after him. The directions for preparing sarcolactic acid, on pp. 442–3, would be improved by the substitution of the word "phosphoric" for "sulphuric," and the method, on p. 426, for decomposing proteids contains more than one serious error, and should be re-written throughout.

But besides these smaller matters, there is an authoritative method adopted of disposing summarily of controversial points by *ex cathedrâ* utterances; we think that a text-book writer might, in a fairly complete work such as this, at least mention the possibility of different views being held by other physiologists, absurd though this may seem to him.

It is a pity that an index was not included in the book;

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